

Pennekamp, F., A. C. Iles, J. Garland, G. Brennan, U. Brose, U. Gaedke, U. Jacob, P. Kratina, B. Matthews, S. Munch, M. Novak, G. M. Palamara, B. C. Rall, B. Rosenbaum, A. Tabi, C. Ward, R. Williams, H. Ye, and O. L. Petchey. 2019. The intrinsic predictability of ecological time series and its potential to guide forecasting. *Ecological Monographs*.

Appendix S1

WPE of Ricker model with lognormal process and observational noise

In the main text, we used simulations of the Logistic Map to evaluate how stochasticity (noise) affects weighted permutation entropy and forecast error. We did so by adding random values from Gaussian distributions. Here we demonstrate the effects of multiplicative lognormally distributed noise. We applied the multiplicative lognormal noise to simulations of the Ricker model:

$$x_{t+1} = x_t * e^{r(1-\frac{x_t}{K})},$$

which produces a wide range of dynamical behavior dependent on the growth rate, r , like the logistic map, but without an upper bound of 1. Our methods closely follow what we did with the logistic map, with the following alterations. We simulated the Ricker model from $r = 2.4$ to 3.4 to encompass the full range of dynamical behavior from simple limit cycles to chaos. Both observational noise and process noise were added (independently) to the simulated population sizes by drawing random values from lognormal distributions with expected standard deviations of either 0, 0.001, 0.01, or 0.1 (Hilborn and Mangel 1997). Finally, we set the carrying capacity, K , to 1000.

In comparison to the logistic map with additive Gaussian noise, the Ricker model with multiplicative lognormal noise produced a very similar relationship between weighted permutation entropy and forecasting error (Appendix S1: Fig. S1, S2). As with the logistic map, both WPE and FE increase in comparison to the no-noise estimates when the dynamics are complex and simpler dynamics are sensitive to the statistical artifact that distorts estimates of WPE (see Appendix S2). Thus, the expected relationship between WPE and FE is robust to multiplicative lognormal noise.

References

Hilborn, R. and Mangel, M. 1997. *The Ecological Detective: Confronting Models with Data*. Princeton University Press, Princeton.

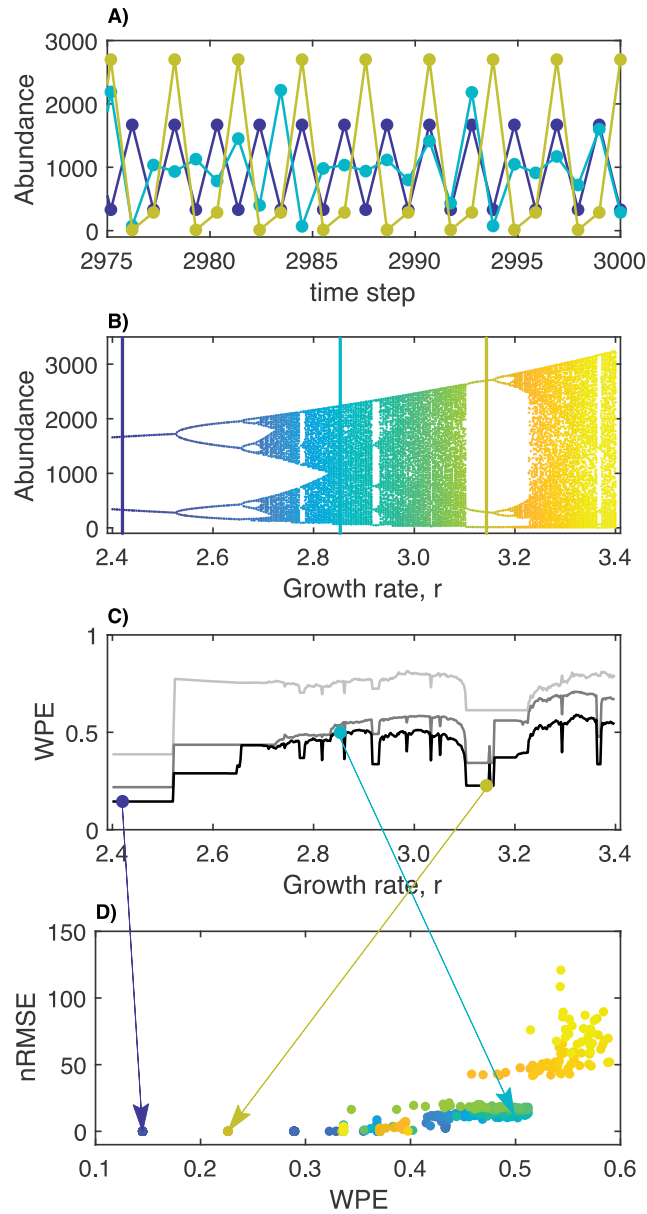


Figure S1: Simulations of the deterministic Ricker model with no added process or observation noise. A) The last 25 time steps of three times series are plotted to demonstrate different behaviors, including 2-point limit cycles ($r \approx 2.42$; dark blue), chaotic behavior ($r \approx 2.85$; turquoise), and 3-point limit cycles within the chaotic realm ($r \approx 3.14$; green). B) A bifurcation diagram of the logistic map attractor for growth rates between $r = 2.4$ and 3.4 . C) Weighted permutation entropy (WPE) of the logistic map time series as the growth rate, r , changes for permutation order, m , of 3 (light grey), 4 (dark grey) and 5 (black), and time delay, τ , of 1. D) forecast error quantified by the normalized root mean squared error (nRMSE) of an EDM forecast ($E = 2$, $\tau = 1$) of the last 200 time steps of each simulation plotted against WPE ($m=5$, $\tau=1$). Marker colors correspond to the growth rates in 'B'.

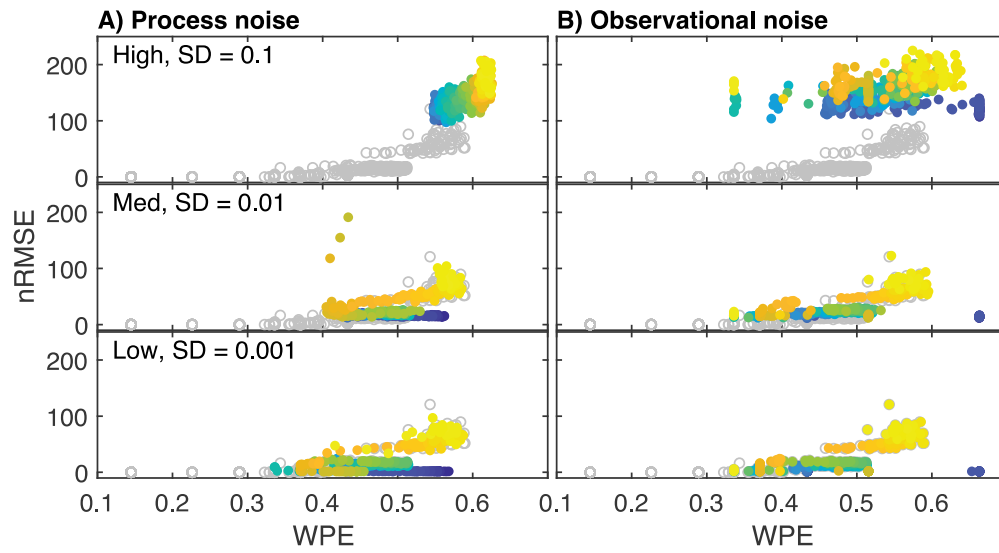


Figure S2: The relationship between weighted permutation error (WPE; $m=5$, $\tau=1$) and forecasting error (measured as nRMSE) at three levels of A) process noise and B) observational noise. In each subplot, the ‘no noise’ case is plotted in grey as a visual reference. The marker colors correspond to the growth rate.

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Appendix S2

Estimating WPE: Choice of τ and the word length m

Here we explain in more detail the counterintuitive result illustrated in Fig. 5 in the main text and Appendix S1: Fig. S2 that simple limit cycle dynamics demonstrate elevated estimates of weighted permutation entropy (WPE) with added noise whereas WPE is robust to noise when dynamics are chaotic. This effect is due to the combined influence of periodicity in the data, temporal sampling with $\tau > 1$ and the chosen word-length, which can cause aliasing effects that distort estimates of WPE. Figure S1 demonstrates the sensitivity of WPE estimates of the logistic map time series at particular growth rates and the effect of the choice of word length and τ as levels of observational noise are increased.

Ideally, WPE estimates would be robust to added noise. However, WPE estimates deviate substantially from the no-noise case when dynamics are 2- or 4-cycle periodic. When the dynamics are periodic, the resulting words contain identical values that result in tied ranks *if there is no added noise*. With added noise, how the ties are broken becomes fully stochastic rather than dependent on the dynamics and the level of noise. This results in an aliasing effect. Note where the no-noise and noise > 0 lines converge as r increases. For $\tau = 1$, this occurs once the logistic map hits period 4 ($r > 3.44949$, dashed line in the figure) for word-lengths 3 and 4: when the dynamics are still a 2-cycle, there are tied ranks in the word, and noise induces a higher WPE; the effect disappears with a 4-cycle. With a word length of 5, however, you have to go past the 4-cycle ($r > 3.54409$, dashed line in the figure) before there aren't any ties that are broken by the additive noise. For reference, figure 4 in the main paper uses the results from the bottom-left panel of figure S1 ($m = 5$, $\tau = 1$).

Similarly, when the dynamics converge to a 3-cycle ($r \sim 3.8284$, dotted line in the figure), there is a strong dip in WPE in nearly all instances, except where $\tau = 3$ and noise > 0 . When τ is set to 3 and the dynamics are 3-cycle, the words consist of identical values, so WPE drops to 0 in the no-noise setting, but is maximal otherwise because the word is completely determined by noise.

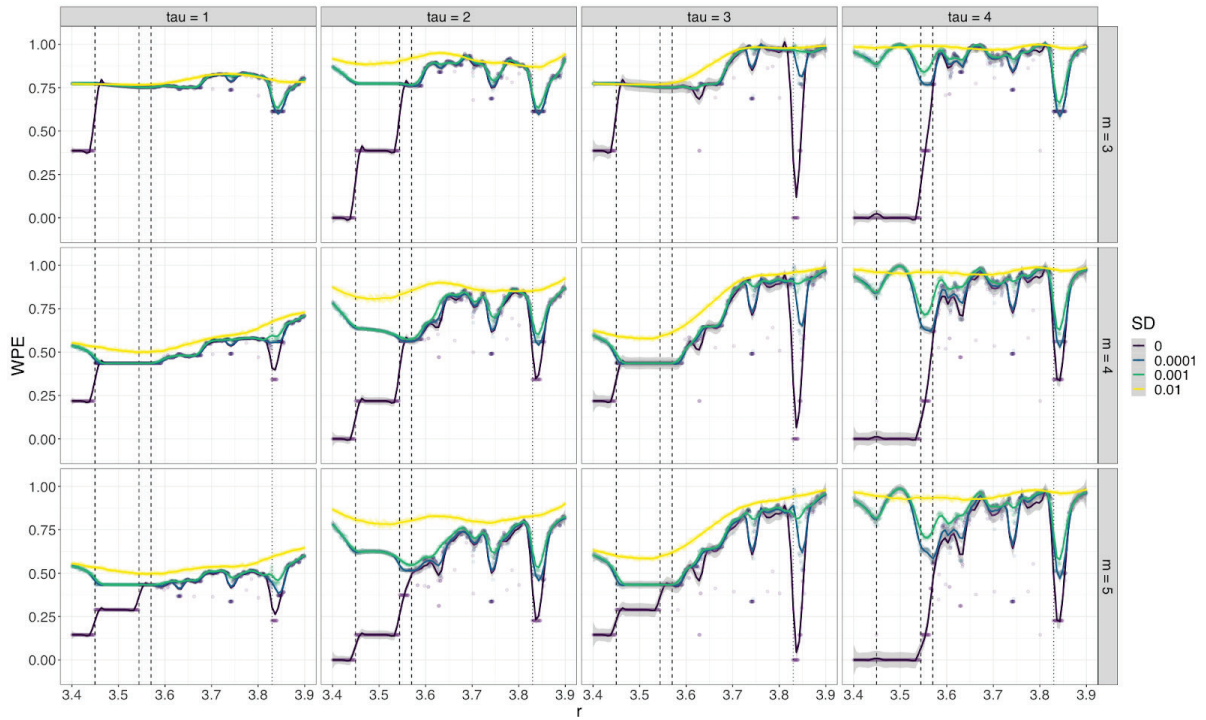


Figure S1: Weighted permutation entropy (WPE) of the logistic map time series as the growth rate, r , changes with no added noise and 3 levels of added observational noise (color). The subplots demonstrate the effects of the chosen time delay used to sample the time series, $\tau=1$ to 4 (columns) and the chosen word length or permutation order, $m=3$ to 5 (rows). The three vertical dashed lines in each subplot indicate where the logistic map transitions from a 2- to 4-limit cycle, then to higher period-doubling, then to chaos and the vertical dotted line on the right indicates where the dynamics converge to a 3-cycle.