Supplemental Material: Annu. Rev. Ecol. Evol. Syst. 2016. 47:409–32
doi: 10.1146/annurev-ecolsys-032416-010215
Characterizing Species Interactions to Understand Press Perturbations: What Is the Community Matrix?
Novak et al.

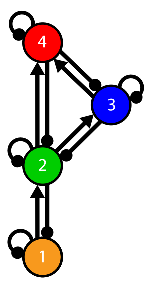
Supplemental Table 1. Glossary of symbols in the general order of appearance.

Symbol	Interpretation
$\mathbf{A}$	A matrix representing an interaction network whose interactions are quantitatively specified. (Used to denote an Alpha, Interaction, Community or, more generally, Jacobian matrix depending on context.)
$^\circ\mathbf{A}$	A qualitatively-specified matrix representation of an interaction network.
$-\mathbf{A}^{-1}$	The negative of the inverse of $\mathbf{A}$, herein referred to as the Net Effects matrix.
$\mathrm{adj}(\mathbf{A})$	A matrix whose elements reflect the net number of positive and negative feedback loops that link each species pair (a.k.a. the classical adjoint).
$\det(\mathbf{A})$	A scalar value contributing equally to all elements of the matrix inverse that modulates the magnitude of species net responses.
α_{ij}	The per capita effect of species j on i as a fraction of species i's intraspecific per capita effect as encapsulated by the Alpha matrix. (Also the corresponding parameter of the Lotka-Volterra competition model.)
a_{ij}	The per capita effect of species j on species i per capita growth rate as encapsulated by the Interactions matrix. (Also the corresponding parameter of the generalized Lotka-Volterra model.)
$a_{jj}^{(-1)}$	The ij^{th} element of the $-\mathbf{A}^{-1}$.
N_i	Species i's abundance.
$\vec{N}$	A vector of species abundances.
$f_i(\vec{N})$	A function describing how species i's *per capita* growth rate, $\frac{1}{N_i}\frac{dN_i}{dt}$, depends on the abundances of the species with whom it interacts, including itself.
$F_i(\vec{N})$	A function describing how species i's *population* growth rate, $\frac{dN_i}{dt}$, depends on the abundances of the species with whom it interacts, including itself.
r_i	Species i's intrinsic per capita growth or death rate.
e	The efficiency by which consumed prey are converted to predator individuals.
K_i	Species i's carrying capacity ($K_i = \frac{r_i}{a_{ii}}$).
c_{ij}	The per capita attack rate of predator j on prey i.
h_{ij}	The handling time associated with the consumption of prey i by predator j.
β_{ij}	The per capita effect of species j on i as a fraction of species j's intraspecific per capita effect as encapsulated by the Beta matrix of Vandermeer (1975).
p	Any to be perturbed factor (parameter or variable) that affects the growth rate of any number of species in a community.
$\check{\mathbf{A}}^{-1}$	The herein defined Normalized Net Effects matrix.
$\check{a}_{ij}$	The ij^{th} element of the Normalized Net Effects matrix.

Supplemental Table 2. Glossary of key terms in the general order of appearance.

Term	Interpretation
Pulse perturbation	Acute, short-term disturbances to one or more species of a community.
Press perturbation	Chronic, long-term disturbances to one or more species of a community.
Interaction modification	An effect by which species alter each others interactions rather than densities (Wootton 1994). Often referred to as an indirect effect in the literature.
Steady state	When long-term average abundances are unchanging in time, including fixed point (constant), oscillatory (e.g. limit cycle) dynamic equilibria.
Qualitative indeterminacy	When a species' true response – increase, decrease or lack of change – cannot be determined (is sign-indeterminate) based on network structure alone.
Apparent mutualism	A positive effect between two species mediated by their interactions with a third species.
Asymptotic stability	The condition whereby the size of a pulse perturbation eventually declines toward zero at a rate given by the leading eigenvalue λ_1.
Hydra effect	When increasing a species' per capita mortality rate causes a net increase in its abundance (Abrams 2009)

a



b

$$\frac{dN_1}{dt} = I + (r_1 + a_{12}N_2)N_1 \qquad \frac{dN_3}{dt} = (r_3 + a_{32}N_2 + a_{33}N_3 + a_{34}N_4)N_3$$

$$\frac{dN_2}{dt} = (r_2 + a_{21}N_1 + a_{22}N_2 + a_{23}N_3 + a_{24}N_4)N_2 \qquad \frac{dN_4}{dt} = (r_4 + a_{42}N_2 + a_{43}N_3 + a_{44}N_4)N_4$$

c

$$\begin{bmatrix} -1 & -4.213 & 0 & 0 \\ 6.667 & -1 & -66.667 & -66.667 \\ 0 & 6.667 & -1 & -66.667 \\ 0 & 3 & 6.667 & -1 \end{bmatrix}$$

d

$$\begin{bmatrix} -2.873 & -0.581 & 0.667 & -5.709 \\ 0.919 & 0.138 & -0.158 & 1.355 \\ -0.399 & -0.060 & 0.071 & -0.738 \\ 0.098 & 0.015 & -0.002 & 0.147 \end{bmatrix}$$

e

$$\begin{bmatrix} -0.237 & -1 & 0 & 0 \\ 0.1 & -0.015 & -1 & -1 \\ 0 & 0.1 & -0.015 & -1 \\ 0 & 0.045 & 0.1 & -0.015 \end{bmatrix}$$

f

$$\begin{bmatrix} -12.103 & -38.728 & 44.437 & -380.610 \\ 3.873 & 9.192 & -10.548 & 90.342 \\ -1.681 & -3.990 & 4.728 & -49.188 \\ 0.412 & 0.979 & -0.126 & 9.772 \end{bmatrix}$$

g

$$\begin{bmatrix} -0.77 & -3.245 & 0 & 0 \\ 0.067 & -0.01 & -0.670 & -0.670 \\ 0 & 0.02 & -3.01 \times 10^{-3} & -0.2 \\ 0 & 6.31 \times 10^{-4} & 1.40 \times 10^{-3} & -2.10 \times 10^{-4} \end{bmatrix}$$

h

$$\begin{bmatrix} -3.729 & -57.774 & 221.677 & -27136.300 \\ 1.193 & 13.713 & -52.618 & 6441.120 \\ -0.518 & -5.952 & 23.584 & -3506.960 \\ 0.127 & 1.461 & -0.627 & 696.716 \end{bmatrix}$$

i

$$\begin{bmatrix} 1 & -4.213 & 9.400 & -38.949 \\ -0.320 & 1 & -2.231 & 9.245 \\ 0.139 & -0.434 & 1 & -5.034 \\ -0.034 & 0.107 & -0.027 & 1 \end{bmatrix}$$

j

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

k

$$\begin{bmatrix} 0.667 & -0.333 & 0.333 & 0 \\ 0.333 & 0.333 & -0.333 & 0 \\ 0 & 0 & 0.5 & -0.5 \\ 0.333 & 0.333 & 0.167 & 0.5 \end{bmatrix}$$

l

$$\begin{bmatrix} 1 & -1 & 0.667 & 0 \\ 0.5 & 1 & -0.667 & 0 \\ 0 & 0 & 1 & -1 \\ 0.5 & 1 & 0.333 & 1 \end{bmatrix}$$

Supplemental Figure 1

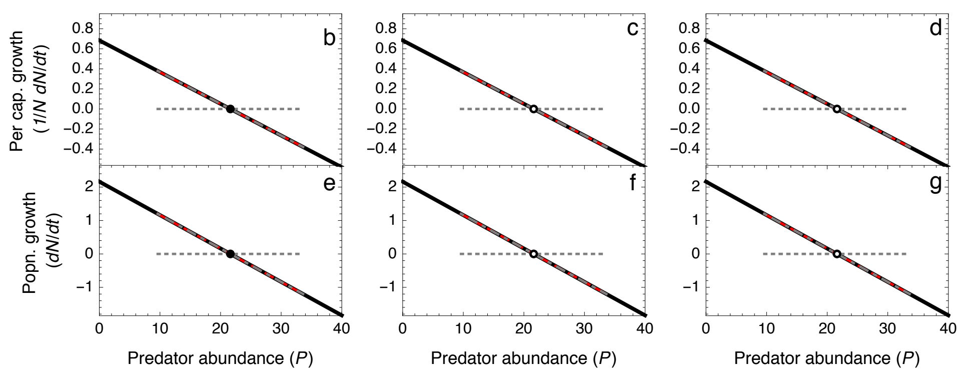
(a) The four-species trophic omnivory system used in the main text to illustrate the utility and assumptions of press perturbation theory. (b) The assumed functional dependencies correspond to the model analyzed by Takimoto et al. (2007) ($I = 2.5^*$, $r_1 = r_2 = -0.1$, $r_2 = r_3^{\dagger} = -0.05$, $a_{12} = a_{23} = a_{34} = -1$, $a_{21} = a_{32} = a_{43} = 0.1$, $a_{42} = 0.045^{\ddagger}$), except that self-limitation is included for all but the basal species ($a_{22} = a_{33} = a_{44} = 0.1$) in order to permit calculation of the Alpha Matrix. (c) The Alpha matrix and (d) its Net Effects matrix. (e) The Interaction matrix and (f) its Net Effects matrix. (g) The Community matrix and (h) its Net Effects matrix. (i) The Normalized Net Effects matrix to which all quantitative Net Effects matrices are scaled. (j) The qualitative 'community matrix' and its associated (k) Net Effects matrix and (l) Normalized Net Effects matrix. All matrices evaluated at steady state. *varied in **Figure 3*a-e***; †varied in **Figure 3*f-j***; ‡as in *Case C* of Takimoto et al. (2007).

a

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - \xi(N)P$$
$$\frac{dP}{dt} = e\xi(N)P - dP$$

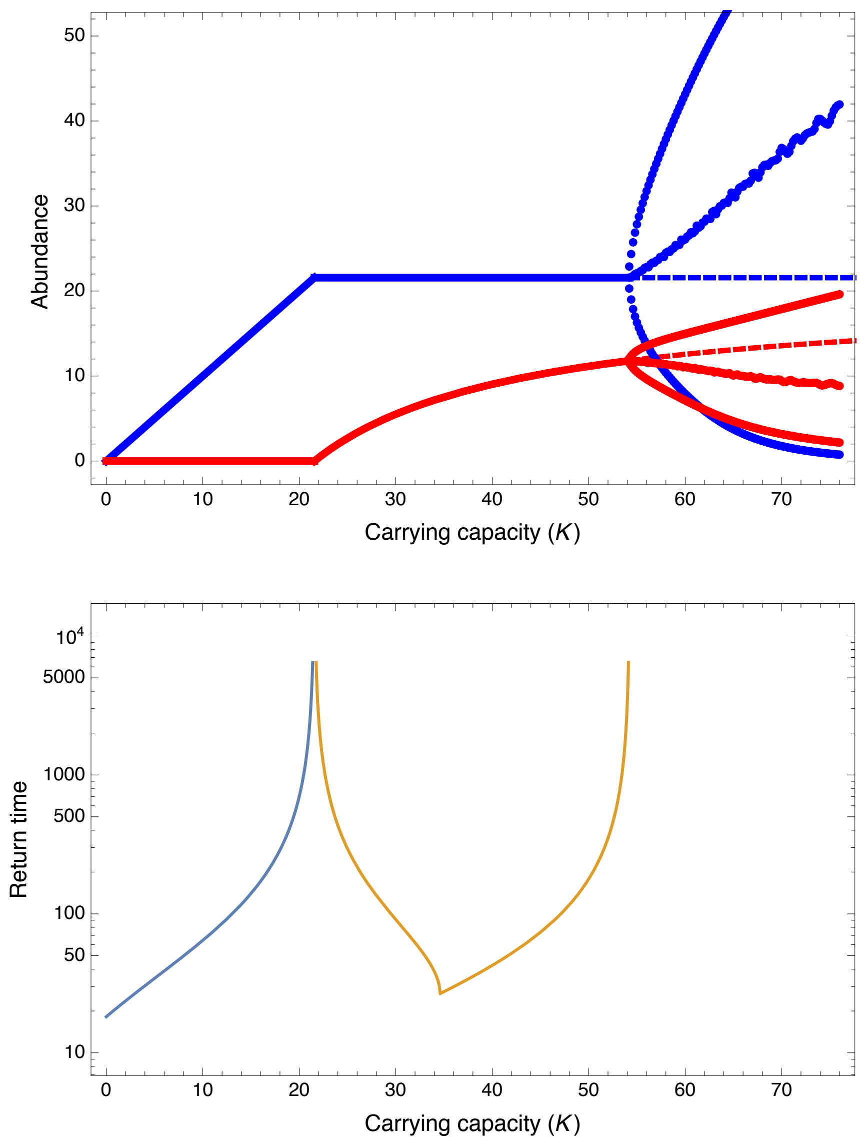
$$\xi(N) = \begin{cases} c_1 N & \text{Type I} \\ \frac{c_2 N}{1+c_2 h N} & \text{Type II} \\ \frac{c_3 N^m}{1+c_3 h N^m} & \text{Type III} \end{cases}$$

$$c_1 = \frac{c_3\phi^{m-1}}{1+c_3 h\phi^m} \qquad c_2 = \frac{ec_3\phi^{m-1}}{(e-dh)\left(1+c_3 h\phi^m\right)} \qquad c_3$$



Supplemental Figure 2

(a) The predator-prey model used in **Figure 3** to illustrate the properties of the Taylor expansion underlying the construction of a Jacobian matrix. Corresponding to **Figure 3**, panels *b*-*g* show how the prey's per capita (*b*-*d*) and population (*e*-*g*) growth rate responds to the predator's population size, P, and how these functions are (perfectly) approximated within the Interaction and Community matrices, respectively. The three scenarios across the columns correspond to the three Holling type functional responses, with the attack rate parameters c_1 and c_2 specified as shown to make the per capita strength of the interspecific effects equal in all cases (as evidenced by the uniformity of all panels). $r = 1$, $K = 10$, $e = 0.1$, $d = 0.01$, $h = 8$, $m = 2$, $c_3 = 0.05$, and $\phi = \frac{\sqrt{d}}{\sqrt{c_3(e-dh)}}$.



Supplemental Figure 3

The transition from a stable (——) to an unstable (- - -) steady state occurs with the emergence of stable limit cycle (minimum, arithmetic mean and maximum abundances shown) in the MacArthur-Rosenzweig paradox of enrichment model (the same predator-prey model used in **Figure 2*b*,*e*** and **Supplemental Figure 2*c*,*f***). Prey in blue, predator in red, with $r = 0.3$, $e = 0.25$, $d = 0.055$, $h = 3.5$ and $c = 0.03$.